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Recent developments in quark nuclear physics

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Abstract. We provide an overview of recent work exploring the quark-mass dependence of hadronic observables and the associated role of chiral non-analytic behavior due to the meson-cloud of hadrons. In particular, we address an issue of great current interest, namely the degree of model independence of results obtained through a controlled extrapolation of lattice QCD simulation results. Physical insights gained from this research are highlighted. We emphasize how chiral effective field theory formulated with a finite-range regulator provides a reliable and *model-independent* extrapolation to the physical world.

PACS. 12.38.Gc Lattice QCD calculations – 11.30.Rd Chiral symmetries – 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes

1 Introduction

Quark nuclear physics describes our attempts to understand the structure of hadronic systems, including nuclei and dense matter, in terms of quarks and gluons —the fundamental degrees of freedom in QCD. It is impossible in just a few pages to provide even a vague outline of the many exciting physics issues currently being addressed in this field —from the possible phase transition to one or more quark-gluon phases at high temperature or density [1], to suggestions of changes of hadron properties in-medium [2,3]. Instead, we shall concentrate on just one development which offers considerable insight into hadron structure from QCD itself, an approach which has led to surprisingly accurate comparisons between lattice QCD data and experiment as well as remarkable insights into how one might improve hadron models.

As the time for calculations within lattice QCD [4] with dynamical fermions (including q-$\bar{q}$ creation and annihilation in the vacuum) scales as $m_q^{-3.6}$ [5], current calculations have been limited to light quark masses 6–10 times larger than the physical ones. With the next generation of supercomputers, around 10 Teraflops, it should be possible to get as low as 2–3 times the physical quark mass, but to actually reach that goal on an acceptable volume will require at least 500 Teraflops. This is probably 10–20 years away.

Since a major motivation for lattice QCD must be to unambiguously compare the calculations of hadron properties with experiment, this is somewhat disappointing. The only remedy for the next decade at least is to find a way to extrapolate masses, form-factors, and so on, calculated at a range of masses considerably larger than the physical ones, to the chiral limit. In an effort to avoid theoretical bias this has usually been done through low-order polynomial fits as a function of quark mass. Unfortunately, as we discuss in sect. 2, this is incorrect and can yield quite misleading results because of the Goldstone nature of the pion.

The essential problem in performing calculations at realistic quark masses (of order 5 MeV) is the approximate chiral symmetry of QCD. Goldstone's theorem tells us that chiral symmetry is dynamically broken and that the non-perturbative vacuum is highly non-trivial [6], with massless Goldstone bosons in the limit $m_q \to 0$. For finite quark mass these bosons are the three charge states of the pion with a mass $m_\pi^2 \propto m_q$. Although this result strictly holds only for m_π^2 near zero (the Gell-Mann–Oakes–Renner relation), lattice simulations show it is a good approximation for m_π^2 up to 1 GeV^2 or so, and we shall use m_π^2 here as a measure of the deviation from the chiral limit.

On these very general grounds, one is therefore compelled to incorporate the non-analyticity into any extrapolation procedure. The classical approach to this problem is chiral perturbation theory (χpt), an effective field theory built upon the symmetries of QCD [7]. There is considerable evidence that the scale naturally associated with chiral symmetry breaking in QCD, $\Lambda_{\chi\mathrm{SB}}$, is of order $4\pi f_\pi$, or about 1 GeV. χpt then leads to an expansion in powers of $m_\pi/\Lambda_{\chi\mathrm{SB}}$ and $p/\Lambda_{\chi\mathrm{SB}}$, with p a typical momentum scale for the process under consideration. At $\mathcal{O}(p^4)$, the corresponding effective Lagrangian has only a small

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number of unknown coefficients which can be determined from experiment. On the other hand, at $\mathcal{O}(p^6)$ there are more than 100 unknown parameters [8], far too many to determine phenomenologically.

While this situation seems formidable, the resolution is already in hand. We must first realize that the lattice data obtained so far represents a wealth of information on the properties of hadrons within QCD itself over a range of quark masses. Just as the study of QCD as a function of N_c has taught us a great deal, so the behaviour as a function of m_q can yield considerable insight into hadronic physics.

The first thing that stands out, once one views the data as a whole, is just how smoothly every hadron property behaves in the region of large quark mass. In fact, baryon masses behave like $a + b\, m_q$, magnetic moments like $(c + d\, m_q)^{-1}$, charge radii squared like $(e + f\, m_q)^{-1}$ and so on. Thus, if one defined a light "constituent quark mass" as $M \equiv M_0 + \tilde{c}\, m_q$ (with $\tilde{c} \sim 1$), one would find baryon masses proportional to M (times the number of u and d quarks), magnetic moments proportional to M^{-1} and so on —just as in the constituent quark picture. There is little or no evidence for the rapid non-linearity associated with the branch cuts created by Goldstone boson loops.

Over the past few years we have come to a deep understanding of why QCD exhibits these features. It will be helpful to summarise those conclusions here:

- In the region of quark masses $m_q > 60$ MeV or so (m_π greater than typically 400–500 MeV) hadron properties are smooth, slowly varying functions of something like a constituent quark mass, $M \sim M_0 + c\, m_q$ (with $c \sim 1$).
- Indeed, $M_N \sim 3\,M, M_{\rho,\omega} \sim 2\,M$ and magnetic moments behave like $1/M$.
- As m_q decreases below 60 MeV or so, chiral symmetry leads to rapid, non-analytic variation, with $\delta M_N \sim {m_q}^{3/2}, \delta\mu_H \sim {m_q}^{1/2}$ and $\delta\langle r^2\rangle_\mathrm{ch} \sim \ln m_q$.
- Chiral quark models like the cloudy bag [9–11] provide a natural explanation of this transition. The scale is basically set by the inverse size of the composite source, above which chiral loops are strongly suppressed. Below this scale, the pion Compton wavelength is larger than the source and one begins to see rapid, non-analytic chiral corrections.

These are remarkable results that will have profound consequences for our further exploration of hadron structure within QCD as well as the analysis of the vast amount of data now being taken concerning unstable resonances. In terms of immediate results for the structure of the nucleon, we note that the careful incorporation of the correct chiral behaviour of QCD into the extrapolation of its properties calculated on the lattice has produced:

i) The most accurate values of the proton and neutron magnetic moments from lattice QCD [12].
ii) The most accurate value of the sigma commutator from lattice QCD [13].
iii) The most accurate values for the charge radii of the baryon octet from lattice QCD [14].
iv) The most accurate values for the magnetic moments of the hyperons from lattice QCD [15,16].
v) Good agreement between the extrapolated moments of the non-singlet distribution, $u-d$, calculated in lattice QCD and the experimentally measured moments [17,18].
vi) The most accurate estimates of the low moments of the spin-dependent parton distribution functions at the physical quark mass from lattice QCD [19].
vii) An understanding of the failures of the assumption of universality of quark electromagnetic properties [20] and an improved lattice estimate of the strangeness magnetic moment of the proton G_M^s [21].

Furthermore, this approach, together with the observed constituent-quark–like behaviour seen in the lattice data for $m_q > 50$ MeV (as noted earlier), has suggested a novel way of modelling hadron structure [22,23].

Apart from the original publications, these developments have been fairly widely reported at various conferences —*e.g.* see refs. [24,25]. Here we focus particularly on the question of the extrapolation of hadron masses in order to clarify an issue of great current interest, namely the degree of model independence of the results obtained after a controlled chiral extrapolation.

2 Effective field theory

Chiral perturbation theory is a low-energy effective field theory for QCD. Low-energy properties of QCD can be expanded about the limit of vanishing momenta and quark mass. In relation to the extrapolation of lattice data, χpt provides a functional form applicable in the limit $m_\pi \to 0$.

Goldstone boson loops give rise to specific corrections to baryon properties —most importantly, they give rise to non-analytic behaviour as a function of quark mass. The low-order, non-analytic contributions arise from the pole in the Goldstone boson propagator and hence are *model-independent* [26]. Analytic variation of hadron properties is not constrained via the symmetry and hence expansions contain free parameters which must be determined by comparison with data.

Effective field theory then tells us that the general expansion of the nucleon mass about the $SU(2)$ chiral limit is

$$m_N = \alpha_0 + \alpha_2\, m_\pi^2 + \alpha_4\, m_\pi^4 + \sigma_{N\pi}(m_\pi, \Lambda) + \sigma_{\Delta\pi}(m_\pi, \Lambda) + \dots, \tag{1}$$

where $\sigma_{B\pi}$ is the self-energy arising from a $B\pi$ loop ($B = N$ or Δ). The expansion has been written explicitly in this form to highlight that the theory is equivalently defined for an arbitrary regulator —see ref. [27] for a complete discussion.

The traditional approach within the literature is to use dimensional regularisation to evaluate the self-energy integrals. Under such a scheme, the $NN\pi$ contribution simply becomes $\sigma_{N\pi}(m_\pi, \Lambda) \to c_\mathrm{LNA}\, m_\pi^3$ and the analytic terms, $\alpha_n\, m_\pi^n$, undergo an infinite renormalisation. The Δ contribution behaves similarly, producing a logarithm and one obtains a series expansion of the nucleon mass about $m_\pi = 0$:

$$m_N = c_0 + c_2\, m_\pi^2 + c_4\, m_\pi^4 + c_{\rm LNA}\, m_\pi^3 + c_{\rm NLNA}\, m_\pi^4 \ln m_\pi + \dots\,, \quad (2)$$

where the α_i have been replaced by the renormalised (and finite) parameters c_i.

It is not clear, *a priori*, that any such truncated expansion will be capable of reliably fitting lattice data. The first empirical indication of serious problems in this approach came with the realization that lattice data could not recover the *model-independent* coefficient, $c_{\rm LNA}$. Truncating the power series at the m_π^3 term and allowing $c_{\rm LNA}$ to vary as a free fit parameter, together with c_0 and c_2, produced a value $c_{\rm LNA} \sim -0.76$ GeV^{-2} [28]. This should be compared with the physical value of -5.6 GeV^{-2} —a factor of 8 larger! This tells us immediately that *either* there are serious convergence problems with the third-order expansion *or* lattice QCD is in error. Clearly, most readers would opt for the first possibility and so do we.

Even by retaining all terms as described in eq. (2), it is not clear that reliable extrapolation can be guaranteed by fitting lattice data over a range of (heavy) quark masses. One point of issue is that it is derived in the limit $m_\pi \ll \Delta(\equiv m_\Delta - m_N)$, whereas the lowest lattice data with dynamical fermions that one can expect in the next decade is perhaps 200–250 MeV —cf. $\Delta = 292$ MeV. It should be clear to those familiar with lattice simulations that even at this lightest pion mass the branch cut will not be observed due to the restricted phase space on a finite-volume lattice. Consequently, all lattice data will still lie above Δ. Mathematically, the region $m_\pi \approx \Delta$ is dominated by a square-root branch cut which starts at $m_\pi = \Delta$. Using dimensional regularisation this takes the form[29]

$$\frac{6g_A^2}{25\pi^2 f_\pi^2}\Big[(\Delta^2 - m_\pi^2)^{\frac{3}{2}} \ln(\Delta + m_\pi - \sqrt{\Delta^2 - m_\pi^2}) - \frac{\Delta}{2}(2\Delta^2 - 3m_\pi^2)\ln m_\pi\Big]\,, \quad (3)$$

for $m_\pi < \Delta$, while for $m_\pi > \Delta$ the first logarithm becomes an arctangent. No serious attempt has been made to extend the formal expansion in eq. (2) to incorporate this cut in an analysis of lattice data and, given the number of parameters to be determined if one works to order m_π^6, it is not likely that it will be done in the next decade.

Even ignoring the $\Delta\pi$ cut for a short time, studies of the formal expansion of the $N \to N\pi \to N$ self-energy integral, $\sigma_{N\pi}$, suggest that it has abysmal convergence properties. Using a sharp, ultra-violet cut-off, Wright showed [30] that the series diverged for $m_\pi > 0.4$ GeV. If one instead uses a dipole cut-off, which in view of the phenomenological shape of the nucleon's axial form-factor is much more realistic, it is worse —with the radius of convergence being around 0.25 GeV. We return to this in sect. 3.

The main issue of the convergence of this truncated series, eq. (2), has its origin fixed in the formalism that it is derived from the general form of eq. (1). The dimensionally regulated approach requires that the pion mass remain much lighter than every other mass scale involved in the problem. This requires that $m_\pi/\Lambda_{\chi\rm SB} \ll 1$ and $m_\pi/\Delta \ll 1$. A further scale, as addressed in the introduction, is set by the physical extent of the source of the pion field. This scale, $\Lambda \sim R_{\rm SOURCE}^{-1}$, corresponds to the transition between rapid, non-linear variation and smooth, *constituent-like* quark mass behaviour. An alternative procedure would be to regulate eq. (1) with a finite Λ which physically corresponds to the source of the meson cloud having an extended structure.

In summary, the low-energy effective field theory can be very useful in describing the quark mass behaviour of hadron properties. These powers have unfortunately been lost in the literature, where only a single type of regulator (*i.e.* dimensional) has been studied in detail.

3 Accurate, model-independent method of chiral extrapolation

We now turn to the direct application of eq. (1), written in a regulator independent form, to the extrapolation of lattice data. By studying a range of different regulators, both finite-ranged, and the dimensional approach, we can study the sensitivity to the form chosen.

The additional analytic term, $\alpha_4\, m_\pi^4$, differs from previous studies using a finite-ranged regulator [28]. Since one is working to non-analytic order $m_\pi^4 \log m_\pi$, one is certainly permitted freedom in α_4 [27]. In practice, whether or not this can be reliably determined, together with Λ, will depend on the data available.

The self-energies, $\sigma_{B\pi}$, are evaluated with a variety of regulators. Included here are both that of the truncated power series resulting from dimensional regularisation and also those of finite range, including sharp cut-off, monopole, dipole and Gaussian. The coefficients of non-analytic terms are constrained to their phenomenological values, while the parameters α_i and Λ are then determined by fitting *lattice data*.

It is again worth noting that, independent of regulator, precisely the same expansion of eq. (2) will be obtained in the limit $m_\pi \ll \Lambda$. Although the parameters α_i *will* depend upon the choice of regulator, one can expand the self-energy terms to order m_π^2 (or higher) to obtain the chiral coefficients at the appropriate order and compare with the truncated expansion of eq. (2) —see ref. [27] for a full discussion of this issue.

The results of our fits to lattice data studied with a range of regulators are shown in fig. 1. The short-dashed curve shows the fit obtained using the dimensionally regulated form of eq. (2) extended to a further term in the expansion, $\alpha_6\, m_\pi^6$. This additional term is necessary for a more reasonable fit due to the large NLNA contribution. The long-dashed curve is the result of using a sharp cut-off regulator, while the following three fits (solid curves), *indistinguishable in this plot* correspond to the monopole, dipole and Gaussian regulated forms.

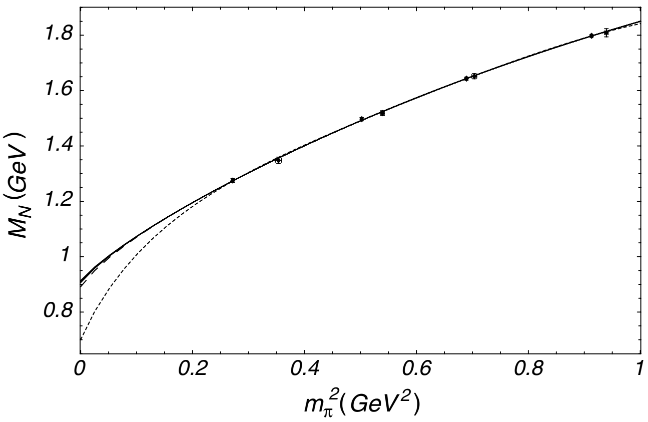

Fig. 1. Fits to lattice data [31] for five different ultra-violet regulators.

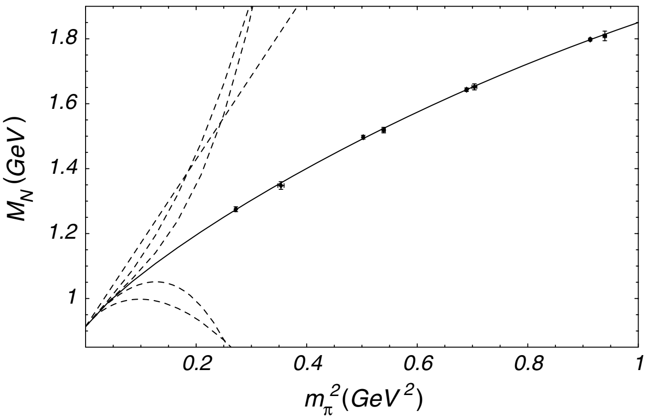

Fig. 2. The fit to the lattice data using the dipole regulator. The dashed curves show power series expansions of this fit to successive orders in m_π for $m_\pi^2 \to m_\pi^6$.

The extrapolation of lattice data is clearly independent of the choice of finite, ultra-violet regulator. Knowing this, we can examine the range of convergence of a truncated power series. Selecting the dipole form and neglecting the $\Delta\pi$ contribution to the nucleon self-energy, one can obtain a closed analytic expression. An expansion in powers of the pion mass is shown in fig. 2. Here we see that convergence of the expansion to order m_π^6 breaks down above $m_\pi \sim$ 250 MeV.

For the reasons outlined, it is essential that the self-energies are evaluated using some ultra-violet regulator —a sharp cut-off or a dipole form, for example. Whatever is chosen does not effect the non-analytic structure which is guaranteed correct. The branch points at m_π equal zero and Δ are incorporated naturally. The use of a finite regulator then automatically produces the transition scale associated with the physical extent of the meson source.

The essential point is that studies of the nucleon (cf. ref. [30] and fig. 1), the Δ (cf. fig. 4 of Leinweber *et al.* [28]) and the ρ meson [32] suggest that *this procedure will result in little or no model dependence in the extrapolation to the physical pion mass once there is accurate lattice data for* $m_\pi \sim 0.3$ GeV *or less*. Physically, this is possible because

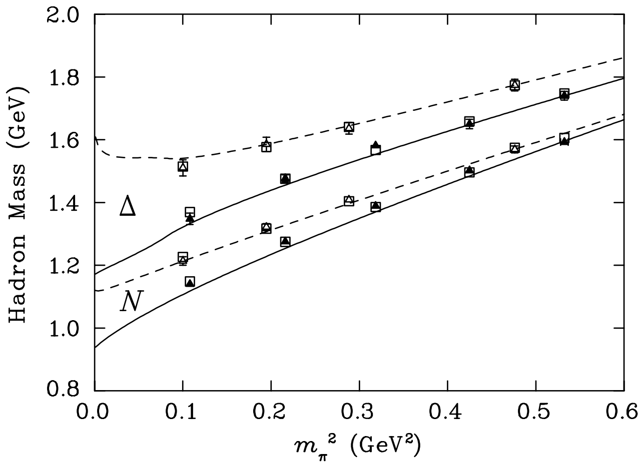

Fig. 3. Fits to both quenched (open triangles) and dynamical (closed triangles) lattice data [33] using a dipole regulator [34].

the self-energy loops are rapidly suppressed in the region $m_\pi > 0.4$ GeV. Thus, an extrapolation based on eq. (1) formulated with the selection of a long-distance regulator allows one to respect *all the chiral constraints*, keep the number of fitting parameters low and yield essentially model-independent results at the physical pion mass. No other approach can do this.

4 Possible connection to QQCD

Although quenched QCD (QQCD) is an unphysical theory, it provides an alternative avenue for enhancing our understanding of chiral extrapolation. Multi-mass techniques allow a dense set of quark masses to be simulated with relative computational ease. Together with recent advances in numerical techniques, which allow simulations to be performed at light quarks masses [35], one will be able to very accurately determine the quark mass dependence of quenched simulations within the light-quark-mass regime.

The study of baryon spectroscopy in quenched lattice QCD has recently made great progress. We have already noted that the lattice data behaves like a constituent quark model for quark masses above 50–60 MeV because Goldstone boson loops are strongly suppressed in this region. This not only provides a very natural explanation of the similarity of quenched and full data in this region but it also suggests a much more ambitious approach to hadron spectra. It suggests that one might remove the small effects of Goldstone boson loops in QQCD (including the η') and then estimate the hadron masses in full QCD by introducing the Goldstone loops which yield the LNA and NLNA behaviour in full QCD.

As a first test of this idea, Young *et al.* [34] recently analysed the MILC data [33] for the N and Δ, using eq. (1) (with $\alpha_4 = 0$) for full QCD and the appropriate generalization for QQCD —*i.e.* using quenched pion couplings as well as the single- and double-"hairpin" η' loops [36,37]. The results illustrated in fig. 3 are remarkable. The values of α_0 and α_2 for the N (or the Δ) obtained in QQCD agree within statistical errors with those obtained in full QCD. Certainly this result (unlike the result for the extrapolation of individual hadron masses as noted above) is somewhat dependent on the shape of the ultra-violet cut-off chosen —although the extent of that is yet to be studied in detail. Nevertheless, given that the study involved the phenomenologically favoured dipole form, it is a remarkable result and merits further investigation.

5 Conclusion

At the present time we have a wonderful conjunction of opportunities. Modern accelerator facilities are providing data of unprecedented precision over a tremendous kinematic range at the same time as numerical simulations of lattice QCD are delivering results of impressive accuracy. It is therefore timely to ask how to use these advances to develop a new and deeper understanding of hadron structure and dynamics.

We have demonstrated that the use of chiral effective field theory can provide accurate extrapolation formulae. In particular, we have shown that the extrapolation of the nucleon mass exhibits minimal model dependence in the choice of finite-ranged, ultra-violet regulator.

In combination with the very successful techniques for chiral extrapolation, lattice QCD will finally yield accurate data on the consequences of non-perturbative QCD. Furthermore, the physical insights obtained from the study of hadron properties as a function of quark mass will guide the development of new quark models and hence a much more realistic picture of hadron structure.

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